SEISMIC ANALYSIS OF BURIED PIPING SYSTEMS

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ABSTRACT

This paper presents a method of seismic analysis for buried piping systems usually encountered in the Nuclear Power Plants. A formulation of seismic forces and moments acting at the pipe bends is developed. This formulation, based on the longitudinal soil strains imposed on the pipe through friction, accounts for the end conditions as well as for the length of the straight legs of pipe connected at the bend. Effect of soil surrounding the elbows is evaluated by using energy principles. Determination of forces and moments at anchors is also included. Guidance to practicing engineers is provided in using the formulation presented in this paper for the seismic analysis of buried piping systems.

INTRODUCTION

With the increasing concern over safety of Nuclear Power Plants, piping engineers are faced with the problem of seismic qualification of buried pipes to satisfy the requirements of the ASME Boiler and Pressure Vessel Code (1). A systematic approach to analyzing buried piping systems has been provided by Goodling (2). This approach is based on the static soil-pipe interaction proposed by Shah and Chu (11) and uses the concept of "effective slippage length" along which friction between the soil and the pipe develops. Based on the theory of beams on elastic foundations (5), this slippage length was used in the evaluation of internal forces and moments at pipe bends and branch connections. Goodling has further refined these techniques by including the flexibility of pipe bends (3), (4).

The purpose of this paper is to provide further development in the seismic analysis of buried pipes using the static soil-pipe interaction model. The effect of soil around elbows is determined by using energy principles. A set of general equations for internal forces and moments at bends is derived. It is valid for long as well as short legs of the bend and also accounts for different end conditions of the legs. Expressions for effective slippage length which include the effect of soil around the elbow are derived. Determination of anchor loads induced by the effect of seismic waves propagating in the soil is also included.

ANALYSIS

a) Effect of Soil Surrounding Elbows:

Pipe bends and elbows subjected to bending are more flexible than straight pipes or solid bends. They are subjected to higher stresses because of a flattening of their circular cross-section as shown in Figure 1. ASME Boiler and Pressure Vessel Code (1) accounts for this effect by specifying a flexibility factor and a stress intensification factor. These factors are applicable to bends on above ground pipes and should be modified to account for the effect of soil for use in buried pipes. Soil tends to reduce flattening of the pipe cross-section by exerting a passive pressure acting on opposite segments of approximately 90° angle (6) as shown in Figure 1. This results in a reduction in the flexibility factor and the stress intensification factor.

The effect of soil on elbow flexibility can be approximately evaluated by using the energy principles. The elastic strain energy U_1, stored during deformation of the bend, per unit length of the centre line has been derived in Reference 10 as:

$$U_1 = \frac{\pi r t E}{2R^2}\left[r^2\eta^2 + 3r\eta a_1 + \frac{9}{4}a_1^2 + \frac{1}{4}\sum_{n=1}^{\infty} a_n^2(1-2n)^2 - 2a_n a_{n+1}(2n-1)(2n+3) + a_{n+1}^2(2n+3)^2 + \frac{\beta^2}{12}\sum_{n=1}^{\infty} a_n^2(8n^3-2n)^2\right] \quad (1)$$

where r = mean cross-sectional radius of the curved pipe, in.
t = pipe wall thickness, in.
E = modulus of elasticity for the pipe material, psi.
R = elbow radius, in.
$\beta = \frac{tR}{r^2\sqrt{1-\nu^2}}$
ν = Poisson's ratio
$\eta = \Delta\theta/\theta$ for in-plane bending
θ = bend angle, radians.
a_n = unknown coefficients in the Fourier Series expression for the tangential deformation w_t of the pipe wall given by:

$$w_t = \sum_{n=1}^{\infty} a_n \sin(2n\phi) \quad (2)$$

in which ϕ = location angle measured in radians from the horizontal axis as shown in Figure 1.

For elbows in buried pipes, the above expession should be modified to include the work done by the passive pressure of the soil, U_2:

$$U_2 = -2\int_{-\pi/4}^{\pi/4} \frac{1}{2}\, p(r\,d\phi)\, w_r \quad (3)$$

where, p = passive pressure of the soil = $k_o w_r$ lbs/in^2
k_o = coefficient of subgrade reaction, lbs/in^3
w_r = radial deformation of the pipe wall at an angular location ϕ, in.

Applying the condition of inextensibility of the elbow centreline in transverse direction, $w_r = -\partial w_t/\partial\phi$, and substituting Equation 2 into 3, we get:

$$U_2 = \pi r k_0 \sum_{n=1}^{\infty} n^2 \; a_n^2 \tag{4}$$

Using the principle of least work, expression for the total strain energy $U = U_1 - U_2$ can be simplified in the following form for first two terms of the series:

$$U = \frac{EI}{2R^2}\left[1 - \frac{9}{2}\frac{B}{C}\right]\eta^2 \tag{5}$$

where I = moment of inertia of the pipe cross-section, in^4.

$$A = 5 + 6\beta^2 + 4\psi$$
$$B = 17 + 600\beta^2 + 16\psi$$
$$C = AB - 25/4$$

$$\psi = \frac{k_0 R^2}{tE}$$

The following expression for the elbow moment M can be obtained by equating the work done by the moment, $M\eta/2R$, to the minimized energy obtained in Equation 5:

$$M = \frac{EI}{R}\left[1 - \frac{9}{2}\frac{B}{C}\right]\eta \tag{6}$$

The reciprocal of the term in brackets is the flexibility factor k_s for the elbow including the soil effect.

It can be shown that by setting $\psi = 0$, the above equation reduces to the equation derived by von Karman (7) using two terms of the Fourier series. The flexibility factor derived here is compared in Table 1 with the flexibility factor specified by the ASME Code (1) for different sizes of pipes. It is clear that for a given value of the coefficient of subgrade reaction k_0, the effect of the soil is dominant as β is reduced.

b) <u>Internal Forces at Elbow Locations</u>

It has been shown that for a 90° elbow maximum stresses occur when the seismic wave propagates along one of the legs called the pipe leg or P leg (4). In this case, the transverse leg is called the T leg. Each elbow in the buried piping system can have any combination of free or fixed ended P and T legs. The formulation given below is applicable to all of these combinations.

Based on the deformed shape of piping (Fig. 2), the relative displacement between soil and pipe at the elbow end is given by (11):

$$\Delta_1 = \epsilon_m L' - \frac{S_1 L'}{AE} - \frac{f L'^2}{2AE} \tag{7}$$

where ϵ_m = maximum soil strain, assumed constant along L', in/in.

L' = effective slippage length along which friction acts, in.
S_1 = internal force in the pipe direction at the elbow, lb.
f = friction force per unit length between soil and pipe
A = cross-sectional area of the pipe, in^2.

Treating each of the P and T legs as a beam on elastic foundation (5, 9) results in the following equations:

$$\Delta_2 = \frac{2\lambda}{k} C_A' S_2 - \frac{2\lambda^2}{k} C_B' M \tag{8}$$

$$\theta_2 = -\frac{2\lambda^2}{k} C_B' S_2 + \frac{4\lambda^3}{k} C_C' M \tag{9}$$

$$\Delta_1 = \frac{2\lambda}{k} C_A S_1 - \frac{2\lambda^2}{k} C_B M \tag{10}$$

$$\theta_1 = \frac{2\lambda^2}{k} C_B S_1 - \frac{4\lambda^3}{k} C_C M \tag{11}$$

where Δ_2= end displacement of the pipe at the elbow in the transverse direction, in.

θ_1 and θ_2= clockwise angle of rotation at the elbow end for the T and P leg respectively, radians.

S_2= shear force at the elbow end of the P leg, lbs.

M= elbow moment, in-lb.

k= soil stiffness per unit length of pipe $= k_o D_o$, lb/in.

D_o= outside diameter, in.

λ = characteristic parameter of the system $= \sqrt[4]{\frac{k}{4EI}}$

C_A, C_B, C_C, C_A', C_B' and C_C' = constants which are functions of λ and leg length, modified to reflect leg end condition (See Appendix I).

The deformation Δ_2 is extremely small since it is perpendicular to the direction of wave propagation and therefore can be neglected. Therefore, for a 90° elbow by setting $\eta = \Delta\theta / \frac{\pi}{2}$, equations 6 through 11 can be combined to solve S_1, S_2, M, Δ_1. Thus:

$$S_1 = C_S \Delta_1 \qquad (12)$$

$$S_2 = \frac{\lambda C_B' C_M}{C_A'} \Delta_1 \qquad (13)$$

$$M = C_M \Delta_1 \qquad (14)$$

$$\Delta_1 = \frac{\epsilon_M L' - f L'^2/2AE}{1 + C_S L'/AE} \qquad (15)$$

where
$$C_S = \frac{k}{2\lambda C_A} + \frac{\lambda C_B}{C_A} C_M$$

$$C_M = \lambda C_B \Big/ \left[C_A \left(\frac{\pi k_s R}{2EI} + C_I \right) \right]$$

$$C_I = \frac{2\lambda^3}{k} \left(2C_C' - \frac{C_B'^2}{C_A'} + 2C_C - \frac{C_B^2}{C_A} \right)$$

In the above expressions, the effective slippage lengths L' is yet to be determined.

c) <u>Determination of Slippage Length</u>

For a long straight pipe with free ends, the maximum slippage length L_{max} is derived from the equilibrium of axial forces as (11):

$$L_{max} = \frac{\epsilon_m AE}{f} \qquad (16)$$

For a bend with a long P leg, this equation is modified to account for the force S_1 at the bend.

$$L_m = \frac{\epsilon_m AE - S_1}{f} \qquad (17)$$

here, L_m = maximum slippage length associated with bend, in.

Equations 7, 12 and 17 are combined to eliminate Δ_1 and S_1. This results in a quadratic equation in L_m which has the following solution:

$$L_m = \frac{AE}{C_S} \left[\sqrt{1 + 2 \frac{\epsilon_m C_S}{f}} - 1 \right] \qquad (18)$$

The effective slippage length L' is equal to L_m (Equation 18) in the following three cases:

1) P leg with a free end and length $l_2 \geqslant L_m + L_{max}$.
2) P leg ending in another elbow and with length $l_2 \geqslant 2L_m$.
3) P leg with a fixed end and length $l_2 \geqslant L_m$.

The effective slippage length for short P legs, where the above requirements for l_2 are not satisfied, depends on the end condition as follows:

For a short P leg with a free end, the axial force diagram is shown in Figure 3. The equilibrium of axial forces yields the following equation for the effective slippage length at the elbow end:

$$L' = \frac{fl_2 - S_1}{2f} \tag{19}$$

Using the same approach as that used in deriving Equation 18, the following expression for L' is obtained:

$$L' = \frac{AE}{3}\left[\sqrt{\left(\frac{\epsilon_m}{f} - \frac{l_2}{AE} + \frac{2}{C_s}\right)^2 + \frac{6l_2}{AEC_s}} - \left(\frac{\epsilon_m}{f} - \frac{l_2}{AE} + \frac{2}{C_s}\right)\right] \tag{20}$$

For a short P leg ending in another elbow, the effective slippage length is (2),

$$L' = \frac{1}{2}\ l_2 \tag{21}$$

For short P leg with a fixed end (2),

$$L' = l_2 \tag{22}$$

d) <u>Determination of Stresses</u>

Maximum stresses in a buried piping system due the propagation of seismic waves can be determined by considering the bends and branches of the system. For a tee branch, the equations given by Shah and Chu (15) and Goodling (2) can be used to determine the internal forces.

For an elbow, seismic waves are first assumed to propagate in the direction of one of its legs, then in the direction of the other leg. For each case, the appropriate expression for effective slippage length (Equations 18 and 20 to 22) is used to determine Δ_1 according to Equation 15. Then, the internal forces at the elbow are calculated using Equations 12, 13 and 14. Hetenyi's equations (5) can be used to determine the internal shear and moment at any other point of interest. The maximum axial force in the pipe leg is calculated as:

$$F_m = S_1 + fL' \tag{23}$$

In addition to soil strain, the seismic wave propagation introduces soil curvature $\mathcal{K}$ causing an additional stress in the piping system equal to (8):

$$S_C = E\ D_0 \mathcal{K}/2 \tag{24}$$

Therefore the total longitudinal stress at an elbow is given by:

$$S_E = 0.75\ i\left(\frac{M}{Z} + S_C\right) + \frac{S_1}{A} \qquad (25)$$

where Z = section modulus of pipe, in^3., and i = stress intensification factor as per Reference 1.

The total stress in the straight pipe due to axial forces and soil curvature is:

$$S_S = \frac{F_m}{A} + S_C \qquad (26)$$

e) Determination of Seismic Anchor Loads

It is possible to calculate the maximum seismic anchor loads by considering the following two cases: the case of seismic waves propagating along the anchored leg, and the case of waves propagating in the perpendicular direction.

When the wave is assumed to propagate along the anchored leg, the axial force at the anchor is equal to the maximum axial force in the pipe given by Equation 23. The force S_2 and the moment M at the elbow are used to calculate the shear force and bending moment at the anchor. The following equations can be obtained by adding expressions in Table 7 case 1 and 4 in Reference 9 and simplifying:

$$S_A = C_D' S_2 + 2\lambda C_E' M \qquad (27)$$

$$M_A = C_F' S_2/\lambda + C_D' M \qquad (28)$$

where S_A = shear force at anchor

M_A = moment at anchor

C_D', C_E' and C_F' = constants, depending on λ and l_2, given in Appendix I.

Then, the wave is assumed to propagate perpendicular to the direction of the anchored leg. Let S_1', S_2' and M' denote the shear, axial force and moment at the elbow with respect to the anchored leg. In this case, the axial force at the anchor is equal to S_2. The shear force at the anchor is given by:

$$S_A' = C_D' S_1' + 2\lambda C_E' M' \qquad (29)$$

The moment at the anchor is written as:

$$M_A' = C_F' \frac{S_1'}{\lambda} + C_D' M' \qquad (30)$$

Therefore, the maximum axial force at the anchor is reported as the higher of F_M or S_2', the maximum shear force is the higher of S_A or S_A' and the maximum moment is the higher of M_A or M_A'.

APPLICATION:

The following steps are provided as a guidance for the seismic analysis of buried piping systems:

a) Information Collection: The analyst should have the piping general layout drawings for the given system. In addition, geotechnical information e.g., max. wave velocities, maximum soil velocity and acceleration, coefficient of subgrade reaction k_0 and friction force per unit length f for the soil/pipe interface.

b) Preparation of Data for Analysis: From the layout drawings, identify all the elbows and branch connections. For each of the elbows, identify the leg lengths l_1 and l_2 and their respective end conditions.

c) Analysis: Forces, moments and stresses at all the elbow locations can be evaluated by using the appropriate equations derived in this paper. For branch connections, use of the expressions given in References (2) and (11) is suggested. A computer program based on this paper was written and has been used to solve the following example:

Example: The following information is given for a pipe with a 90° elbow: D_0 = 36 in, E = 27.9×10^6 psi, t = 0.5 in.
R = 54 in., k_0 = 410 lb/in^3, f = 317 lb/in, ϵ_m = 3.33×10^{-4} in/in., $\mathcal{K}$ = 0.15×10^{-6} rad., the length of the P leg = 1160 in.

For illustration purposes, different end conditions for P leg and T leg are considered and for each combination of these conditions, length of the T leg is assumed to be either 63 in. or 252 in.

Results are summarized in Table 2 and it can be seen that the proposed method reflects the effect of length and end conditions of the P and T legs when applicable.

DISCUSSION

The effect of soil on the stress intensification factor for the bends has not been considered here and the use of the ASME Code (1) value is suggested as it is conservative. In the derivation of k_s, only two terms of the Fourier series have been used. The use of more terms leads to tedius and lengthy derivations, and resulting accuracy is not expected to offset the conservatism achieved by neglecting the effect of soil on the stress intensification factor.

REFERENCES

1. "ASME Boiler and Pressure Vessel Code", Section III, American Society of Mechanical Engineers, 1980.

2. Goodling, E.C., "Flexibility Analysis of Buried Pipe", Joint ASME/CSME Pressure Vessels and Piping Conference, Montreal, Canada, June 1978.

3. Goodling, E.C., "Seismic Stress in Buried Elbows", Preprint 3595, ASCE National Convention, Boston, April 1979.

4. Goodling, E.C., "More on Flexibility Analysis of Buried Pipe", ASME Pressure Vessels and Piping Conference, San Fransisco, California, August 1980.

5. Heteny, M., "Beams on Elastic Foundation", the University of Michigan Press, Ann Arbor, Mich., 1967.

6. Hovgaard, W., "The Elastic Deformation of Pipe Bends", Journal of Mathematics and Physics, Vol. 6, No.2, 1926.

7. Karman, T., "Uber die Formanderung duennwandiger Rohre", Zeitschrift des Vereines deuticher Ingenierure, Vol. 55, 1911.

8. Newmark, N.M, "Earthquake Response of Reactor Structures", Proceedings of the First International Conference on Structural Mechanics in Reactor Technology, Berlin, Germany, September 1971.

9. Roark, R.J. and Young, W.C., "Formulas for Stress and Strain", 5th Edition, McGraw-Hill Book Co., New York, 1975.

10. Rodabaugh, E.C., and George, H.H., "Effect of Internal Pressure on Flexibility and Stress-Intensification Factors of Curved Pipe or Welding Elbows", Transactions ASME, Vol. 79, 1957.

11. Shah, H.H. and Chu, S.L., "Seismic Analysis of Underground Structural Elements", Journal of the Power Division, Proceedings of ASCE, Vol. 100, No. PO 1, July 1974.

<u>TABLE 1: EFFECT OF SOIL ON ELBOW FLEXIBILITY FACTOR</u> : k_i^* vs k_s

k_0 = 400 lb/in^3, E= 30 X 10^6 lb/in^2, ν = 0.3, Bend radius R = 3r

Pipe Radius r in.	Pipe Wall thickness t in.	$\frac{t}{r}$	β	ψ	k_i $(0.9/h)$	k_s $\left[1/\left(1-\frac{9\beta}{2c}\right)\right]$
6	0.375	0.0625	0.1965	0.01152	8.804	8.224
8	0.375	0.04688	0.1474	0.02048	11.74	9.89
12	0.375	0.03125	0.0983	0.04608	17.6	10.44
18	0.375	0.02083	0.0655	0.1037	26.4	7.96
26	0.375	0.0144	0.0453	0.2167	38.19	5.124

* k_i = Flexibility factor defined in Ref. (1)
h = Flexibility characteristic = tR/r^2

TABLE 2: RESULTS OF THE EXAMPLE PROBLEM

D_0 = 36 in   ELBOW ANGLE = 90°

P Leg		T Leg		Pipe Stress in Straight Part (Eq. 26), psi	Elbow Stress (Eq. 25), psi
length in.	End Cond.	length in.	End Cond.		
1160	free	63 252	free free	3802 4052	6639 4267
1160	another elbow	63 252	free free	4318 4944	7279 4888
1160	fixed	63 252	free free	7990 8714	10057 6281
1160	fixed	63 252	fixed fixed	9366 8752	8779 6288

APPENDIX I

Mathematical expressions for the constants used in this paper:

l_1 = length of the T leg l_2 = length of the P leg

$C_1 = \operatorname{Sinh}\lambda l_1 \; \operatorname{Cosh}\lambda l_1 - \sin\lambda l_1 \; \cos\lambda l_1$

$C_2 = \operatorname{Sinh}^2\lambda l_1 - \sin^2\lambda l_1,$

$C_3 = \operatorname{Sinh}^2\lambda l_1 + \sin^2\lambda l_1,$

$C_4 = \operatorname{Sinh}\lambda l_1 \; \operatorname{Cosh}\lambda l_1 + \sin\lambda l_1 \; \cos\lambda l_1$

$C_5 = 0$ for a T leg with a free end

$\quad = 2$ for a T leg with a fixed end

$C_6 = \operatorname{Sinh}\lambda l_1 \; \cos\lambda l_1 - \operatorname{Cosh}\lambda l_1 \; \sin\lambda l_1$

$C_7 = 2 \operatorname{Cosh}\lambda l_1 \; \cos\lambda l_1$

$C_8 = \operatorname{Sinh}\lambda l_1 \; \cos\lambda l_1 + \operatorname{Cosh}\lambda l_1 \; \sin\lambda l_1$

$$C_A = \frac{C_1}{C_2+C_5}, \quad C_B = \frac{C_3}{C_2+C_5}, \quad C_C = \frac{C_4}{C_2+C_5}$$

$C_1', C_2', \ldots, C_8', C_A', C_B'$ & C_C' = Constants applicable to P leg obtained by replacing l_1 with l_2 in the above expressions.

$$C_D' = \frac{C_7'}{2+C_2'}, \quad C_E' = \frac{C_6'}{2+C_2'}, \quad C_F' = \frac{C_8'}{2+C_2'}$$

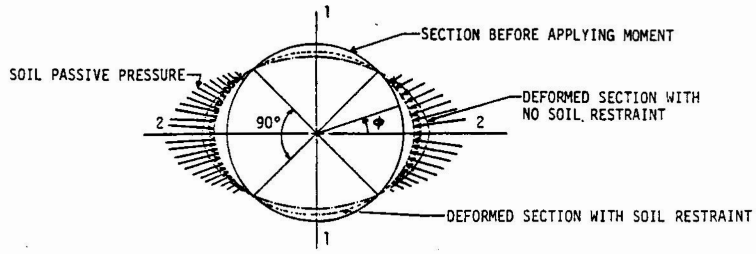

FIG. 1

X-SECTION IN AN ELBOW SUBJECTED TO IN-PLANE BENDING MOMENT

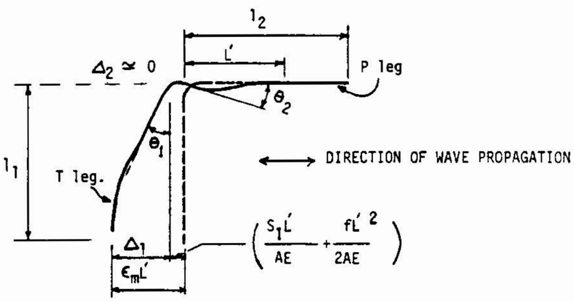

FIG. 2

DEFORMED SHAPE OF BURIED ELBOW AND ATTACHED LEGS

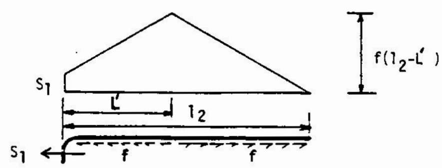

FIG. 3

AXIAL FORCE DIAGRAM FOR A FREE END PIPE ($l_2 < L_m + L_{max}$)